Effects of Soret/Dufour on Steady MHD Mixed Convection over an Infinite Vertical Plate embedded in a Porous Medium with a Convective Boundary Condition

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Abstract

The focus of this paper is to study the effects of Dufour and Soret on steady magnetohydrodynamic mixed convection over an infinite vertical plate with a convective boundary condition embedded in a porous medium in the presence of first-order chemical reaction. The boundary layer equations governing the flow are written into a dimensionless form, which are numerically solved by applying an implicit finite-difference scheme. A discussion is provided for the effect of Dufour number (Du), Soret number (Sr) and Biot number (Bi) on the velocity, temperature and concentration profiles. Numerical results for the local skin-friction coefficient, the local Nusselt number and the local Sherwood number are also illustrated graphically for different physical parameters.

***Keywords**:* MHD flow, Convective boundary condition, Dufour and Soret effects, Porous medium

1. Introduction

Free and forced convection flow on a surface is of substantial importance in nature and engineering processes. Magneto-hydro-dynamic (MHD) describes a class of phenomena that occurs in the flow of electrically conducting fluids with magnetic fields. This type of flow in the presence of chemical reaction has many applications such as geothermal reservoirs, thermal insulation, packed-bed catalytic reactors, cooling of nuclear reactors, underground energy transport and etc. Chemical engineering systems are also frequently occurred in a porous media for controlling transport phenomena, such as radioactive waste disposal, petroleum reservoirs recovery and etc.

The heat and mass transfer can affect each other at the same time. The heat flux due to a concentration gradient is known as the Dufour effect (diffusion-thermal). On the other hand, the Soret effect (thermal-diffusion) is the reciprocal phenomenon, the occurrence of a diffusion flux caused by a temperature gradient. Chapman and Cowling [1] and Hirshfelder et al. [2] have analyzed these effects from the kinetic theory of gases.

More recently, Lin et al [3] have reported combined heat and mass transfer by laminar natural convection from a vertical plate. Hossain et al [4] studied the Radiation effect on mixed convection along a vertical plate with uniform surface temperature. Yin [5] has discussed the effect of free convection on MHD coupled heat and mass transfer of a moving permeable vertical surface. Chamkha et al [6] have considered hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid saturated porous medium. Postelnicu [7] examined the Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. He solved the problem with a constant wall temperature and without considering chemical reaction and suction velocity at the plate. Mansour et al [8] analyzed the effects of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret and Dufour numbers. They did not consider the effect of suction in the model and solved the problem without considering the viscous dissipation.

Recently, Makinde et al [9] presented a convective boundary condition in a MHD mixed convection flow from a vertical plate embedded in a porous medium. They did not investigate the Dufour and Soret effects in their model. The objective of the present paper is to study the effects of Soret and Dufour by considering a convective boundary condition on steady MHD mixed convection over an infinite vertical plate embedded in a porous medium considering a first-order chemical reaction.

2. Mathematical model

The schematic configuration of problem is an infinite vertical plate embedded in a saturated porous medium as shown in Fig. 1. A cartesian coordinate system is used for the problem. The x-axis is considered along the surface and the y-axis is chosen normal to it. It is assumed that a cold Newtonian and electrically conducting fluid on the right side of the surface is viscous and incompressible. The fluid is maintained at temperature T_∞ and all its thermo-physical properties is taken independent of temperature (T) and concentration (C) of chemical species (except for the density (ρ)). The mixed convection flow is assumed to be steady, laminar, and hydro-magnetic coupled heat and mass transfer. The right wall of the vertical plate is exposed to a hot fluid at temperature T_f with a convection heat transfer coefficient of h_f. A homogeneous magnetic field of strength B_0 is forced to the direction of normal to the plate. The induced magnetic field is considered negligible compared to applied field, because of small magnetic Reynolds number for most fluids in engineering processes.

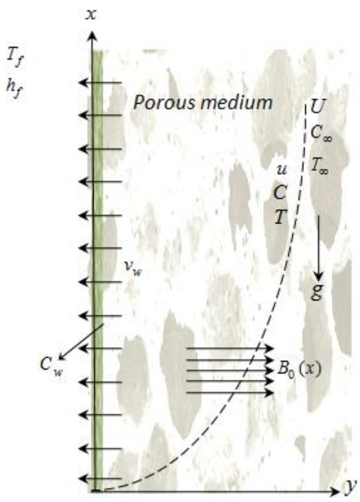

Fig. 1 Schematic of coordinate system

Under the above states and boundary-layer approximation, the momentum, energy and concentration equations governing the flow by considering the Boussinesq approximation and effects of Soret and Dufour can be written as follows [9,10]:

$$-v_w \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_t (T - T_\infty) +$$

$$g\beta_m (C - C_\infty + \frac{\nu}{K}(U - u) + \frac{\sigma B_0^2}{\rho}(U - u) \quad (1)$$

$$-v_w \frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 +$$

$$\frac{\sigma B_0^2}{\rho C_p}(U - u)^2 + \frac{D k_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \quad (2)$$

$$-v_w \frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \gamma (C - C_\infty) + \frac{D k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \quad (3)$$

The boundary conditions for this problem may be given by:

$$y = 0: u = 0; -k_w \frac{\partial T}{\partial y} = h_f (T_f - T); C = C_w \quad (4)$$

$$y \to \infty : u \to U; T \to T_\infty; C \to C_\infty \quad (5)$$

where u is the velocity component along the x-axis, β_t and β_m are thermal and concentration expansion coefficients, ν, k, k_w and C_p are kinematic viscosity, thermal conductivity of fluid, thermal conductivity of solid and specific heat at constant pressure, respectively. σ and K are electrical conductivity and permeability parameter. g is the gravitational acceleration. v_w is the wall suction velocity. Free stream conditions of temperature and concentration is denoted by subscript ∞. U is the free steam velocity. D, k_T, C_s, γ and T_m are mass diffusivity, thermal diffusion ratio, concentration susceptibility, reaction rate coefficient and mean fluid temperature, respectively. Furthermore, C_w is the local concentration at plate surface.

Eqs. (1) – (5) can be written in dimensionless ordinary differential equations by introducing the following dimensionless quantities:

$$\eta = \frac{v_w y}{\nu} \quad (6)$$

$$f = \frac{u}{v_w} \quad (7)$$

$$F = \frac{U}{v_w} \quad (8)$$

$$\theta = \frac{T - T_\infty}{T_f - T_\infty} \quad (9)$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (10)$$

$$Sc = \frac{\nu}{D} \quad (11)$$

$$Gr = \frac{g\beta_t (T_f - T_\infty)\nu}{v_w^3} \quad (12)$$

$$Gc = \frac{g\beta_m (C_w - C_\infty)\nu}{v_w^3} \quad (13)$$

$$\Pr = \frac{\rho \nu C_p}{k} \quad (14)$$

$$\lambda = \frac{\gamma \nu^2}{D v_w^2} \quad (15)$$

$$M = \frac{\sigma B_0^2 \nu}{\rho c} \quad (16)$$

$$Bi = \frac{h_f \nu}{k_w v_w} \quad (17)$$

$$\kappa = \frac{v_w^2 K}{\nu^2} \quad (18)$$

$$Ec = \frac{v_w^2}{C_p (T_f - T_\infty)} \quad (19)$$

$$Du = \frac{D k_T (C_w - C_\infty)}{\nu C_s C_p (T_f - T_\infty)} \quad (20)$$

$$Sr = \frac{D k_T (T_f - T_\infty)}{\nu T_m (C_w - C_\infty)} \quad (21)$$

where η is the dimensionless coordinate along the y-axis. $\Pr$ is the Prandtl number. M is the Hartmann number or magnetic field parameter.

Ec is the Eckert number. Gr and Gc are thermal and mass transfer Grashof numbers. κ is the permeability parameter. Sc and λ are Schmidt number and reaction rate parameter. F is the dimensionless velocity of free stream. f, θ and ϕ are the dimensionless velocity, temperature and concentration, respectively.

Dimensionless form of Eqs. (1) – (5) can be written as:

$$-\frac{df}{d\eta}=\frac{d^2f}{d\eta^2}+Gr\theta+Gc\phi+M(F-f)+\frac{F-f}{\kappa} \quad (22)$$

$$-\frac{d\theta}{d\eta}=\frac{1}{\Pr}\frac{d^2\theta}{d\eta^2}+Ec\left(\frac{df}{d\eta}\right)^2+MEc(F-f)^2+Du\frac{d^2\phi}{d\eta^2} \quad (23)$$

$$-\frac{d\phi}{d\eta}=\frac{1}{Sc}\frac{d^2\phi}{d\eta^2}-\frac{\lambda}{Sc}\phi+Sr\frac{d^2\theta}{d\eta^2} \quad (24)$$

$$\eta=0: f=0; \frac{d\theta}{d\eta}=Bi[\theta-1]; \phi=1 \quad (25)$$

$$\eta\to\infty: f=F; \theta=\phi=0 \quad (26)$$

The physical quantities of interest are the local skin-friction coefficient, the local Nusselt number and the local Sherwood number are defined, respectively, by:

$$C_f=\frac{2}{F^2}\cdot f'(0) \quad (27)$$

$$Nu=\frac{h\nu}{k\nu_w}=-\frac{T_f-T_\infty}{T_w-T_\infty}\cdot\theta'(0) \quad (28)$$

$$Sh=\frac{h_m\nu}{D\nu_w}=-\phi'(0) \quad (29)$$

where T_w and h_m are the plate surface temperature and the convection mass transfer coefficient, respectively.

The quantities of $f'(0)$, $-\theta'(0)$ and $-\phi'(0)$ which are proportional to the local skin-friction coefficient, the local Nusselt number and the local Sherwood number, respectively, are obtained by the numerical solution.

3. Numerical solution

An implicit finite difference scheme of second order has been employed for solving the system of ordinary differential Eqs. (22) – (24) that are highly coupled, with the boundary conditions (Eqs. (25) and (26)). The numbers of grids in numerical domain are chosen 10000 points for reducing the truncation error and the convergence criterion is taken 10^{-8} for lowering the error of termination. In order to ensure the accuracy of the results, a comparison has been carried out for a special case. For $Du=0$ and $Sr=0$, the values of the wall shear stress, and the local Nusselt and Sherwood numbers have been compared with those given by Makinde and Aziz [9] (Table. 1). As shown in Table. 1, a good agreement between the results is found. For this purpose, the values of $\Pr$, Sc and F as those in article are 0.72, 0.24 and 0.5, respectively and also the value of M, Ec, Gr, Gc, κ, λ is 0.1.

Table. 1 Comparison of values of $f'(0)$, $-\theta'(0)$ and $-\phi'(0)$ with $Du=0$ and $Sr=0$.

Bi	Makinde and Aziz [9]			Present results		
	$f'(0)$	$-\theta'(0)$	$-\phi'(0)$	$f'(0)$	$-\theta'(0)$	$-\phi'(0)$
0	1.892	0.000	0.459	1.888	0.000	0.459
0.1	1.895	0.084	0.459	1.891	0.084	0.460
1	1.908	0.398	0.459	1.904	0.398	0.459
10	1.917	0.639	0.459	1.914	0.639	0.459

4. Results and discussion

A cooling problem is considered because of the wide range of its applications such as the cooling of nuclear reactors and electronic components. For the cooling problem, the thermal Grashof number is positive ($Gr > 0$) and the mass transfer Grashof number is also taken positive which shows that the concentration at the plate surface is more than the free stream concentration ($Gc > 0$). All the numerical results are obtained for $\mathrm{Pr} = 0.72$ that corresponds to air and $Sc = 0.24, 0.62, 0.78$ which expresses the diffusion of hydrogen, water and ammonia in air, respectively. Physical properties are shown in Table. 2. Furthermore, a free stream velocity of $F = 0.5$ is chosen for all profiles.

Table. 2 Physical properties of fluid (air) and species (water vapor, hydrogen and ammonia) at $20\ ^\circ C$ and $1\ atm$ [11].

ν , $(\frac{m^2}{s})$	15.11e-6	
k , $(\frac{W}{m.K})$	0.0257	
ρ , $(\frac{kg}{m^3})$	1.205	
C_p , $(\frac{kJ}{kg.K})$	1.005	
β_t , $(\frac{1}{K})$	3..43e-3	
β_m , $(\frac{m^3}{kmol})$ between species x and air	$x = H_2$	11.12
	$x = H_2O(v)$	0.50
	$x = NH_3$	0.89

In Fig. 2 the effects of the magnetic field parameter and Biot number on velocity profiles are depicted. It is observed that the values of velocity increase with Bi due to a drop in thermal resistance of the plate and decrease with M due to exerting an opposite force on the fluid by the magnetic field. Fig. 3 shows that the values of temperature increase by increasing the Biot number, because of high convective heat transfer to the cold fluid. The influence of Biot number on the values of concentration is illustrated in Fig. 4. It is clear that the increase in Biot number increase the concentration.

The velocity, temperature and concentration profiles are observed in Figs. 5 and 6 for different values of thermal and

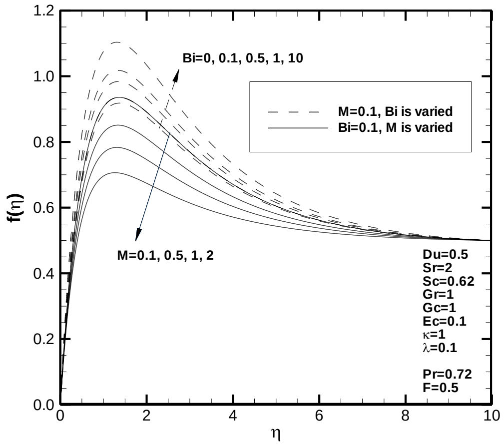

Fig. 2 Velocity profiles for different values of Biot number and magnetic parameter.

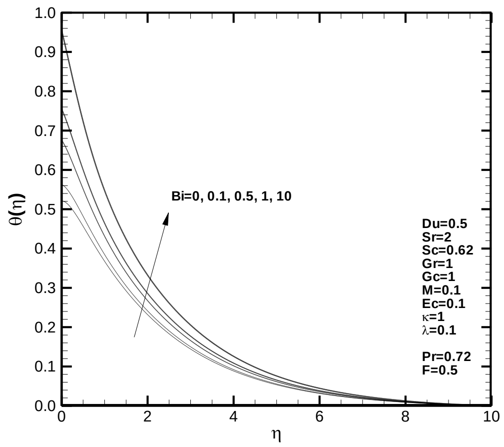

Fig. 3 Temperature profiles for different values of Biot number.

mass transfer Grashof numbers (Gr and Gc), respectively. When Gr and Gc increase, the thermal and mass buoyancy forces increase. Therefore the values of velocity, temperature and concentration increase with Gr and Gc.

To illustrate the effect of permeability parameter, the velocity and temperature profiles are presented in Fig. 7. Both the velocity and temperature increase with increasing the permeability parameter due to retarding effect of porous medium on the flow.

The existence of velocity, heat and mass fluxes into fluid is caused to generate the viscous dissipation. The viscous dissipation is known as heat generation source inside the fluid and consequently should increase the temperature and velocity values. Viscous dissipation effect is presented in Fig. 8 through variation of Eckert number. Fig. 8 shows velocity and temperature profiles for different values of Eckert number. As said as before, velocity and temperature values are enhanced by increasing the Eckert number.

An increase in the Schmidt number

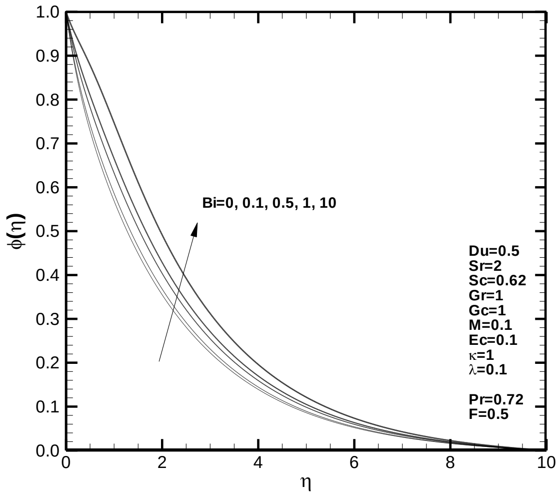

Fig. 4 Concentration profiles for different values of Biot number.

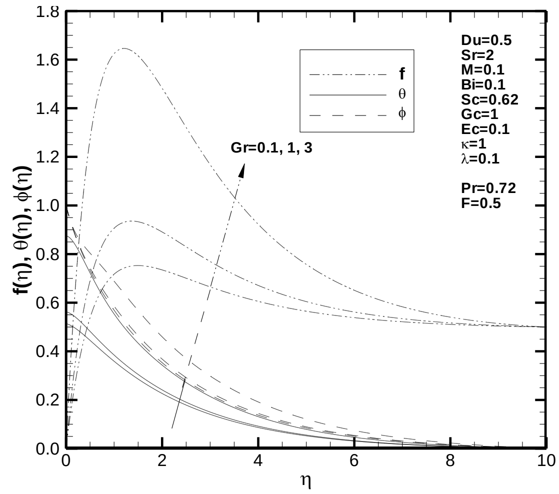

Fig. 5 Effect of Gr on the velocity, temperature and concentration profiles.

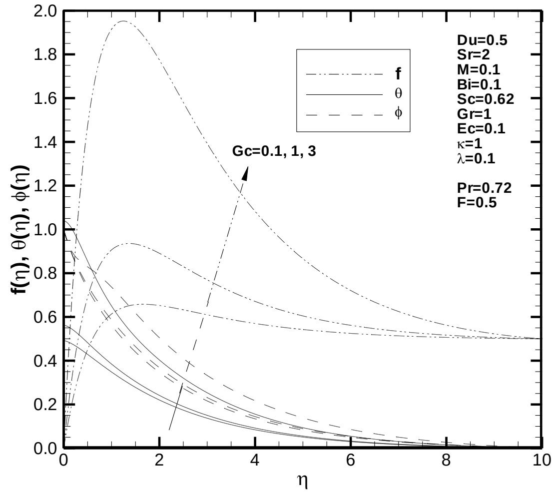

Fig. 6 Effect of Gc on the velocity, temperature and concentration profiles.

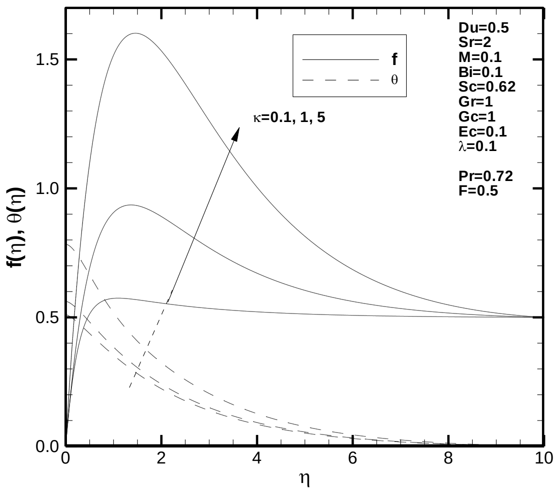

Fig. 7 Effect of κ on the velocity and temperature profiles.

and reaction rate parameter leads to a fall in the concentration (See Fig. 9). The Schmidt number is known as the ratio of momentum diffusivity and mass diffusivity and increasing that is concluded to reduce the concentration values. In high value of reaction rate parameter, the consumption of the species increases, therefore the concentration of the species tends to decrease.

Figs. 10 – 12 are obtained for three cases: $Du = Sr = 0$, $Du/Sr = 0.25$ and $Du/Sr = 4$. From Figs. 10 – 12, it can be found that when Du and Sr increase, the velocity, temperature and concentration increase. As shown in Figs. 10 and 11, the effect of the Soret number on velocity and temperature profiles is less than the Dufour number. In high Dufour numbers, more heat flux is produced by the concentration gradient (Eq. (23)). The value of the concentration is affected much more by the Soret number (See Fig. 12). In high Soret numbers, more mass flux is generated by the temperature gradient (Eq. (24)).

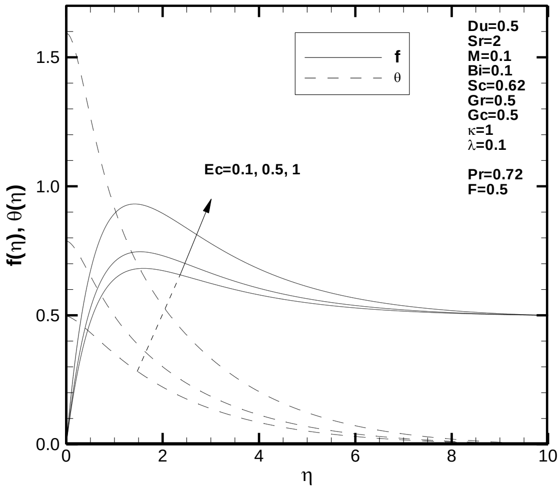

Fig. 8 Effect of Ec on the velocity, temperature and concentration profiles.

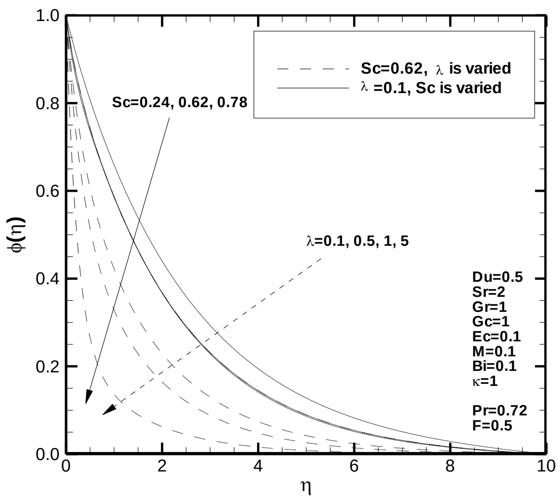

Fig. 9 Effect of Sc and λ on the concentration profiles.

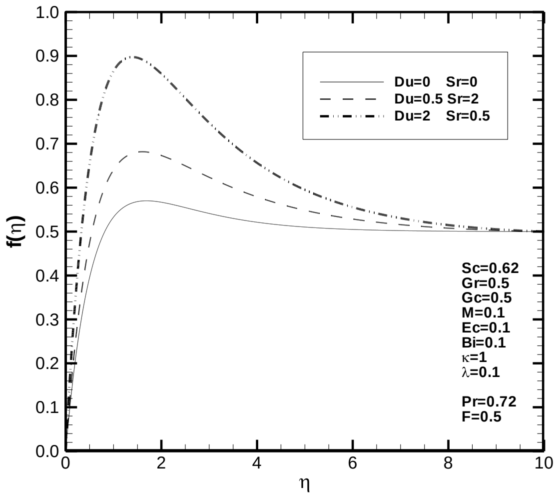

Fig. 10 Effect of Soret and Dufour on the velocity profiles.

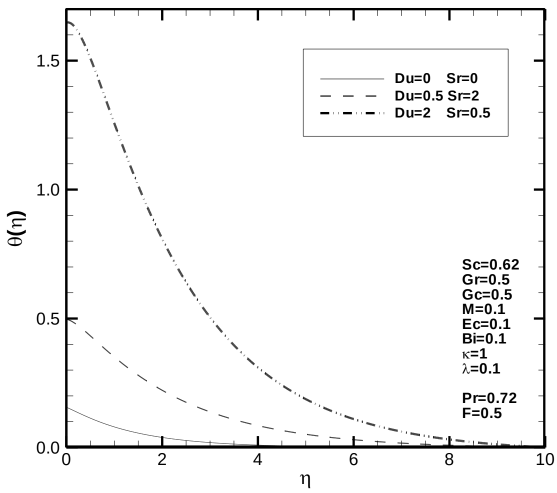

Fig. 11 Effect of Soret and Dufour on the temperature profiles.

The influences of the Dufour and Soret numbers on the local skin-friction coefficient, the local Nusselt number and the local Sherwood number for various values of the Biot number are depicted in Figs. 13 – 18. Figs. 13 – 15 clearly display that $f'(0)$ and $-\phi'(0)$ increase whereas $-\theta'(0)$ decrease with increase in Du. It is seen from Fig. 13 that the skin-friction coefficient is affected by the Biot number and its value is much more for high Bi. Similar effect of the Biot number is noticed on the Nusselt number as shown in Fig 14. For $Bi = 0$, there is no convective heat transfer then the Nusselt number has no changes. Fig. 15 represents a drop in the Sherwood number with the Biot number. Figs. 16 and 17 show that there are no significant effects on the values of the skin-friction coefficient and Nusselt number by increasing the Soret number. However, their values increase with Bi. In Fig. 18, it is observed that the Sherwood number increases with Sr for $Bi = 0$. For the other values of the Biot number, $-\phi'(0)$ decreases with Sr. When the value of the Biot number

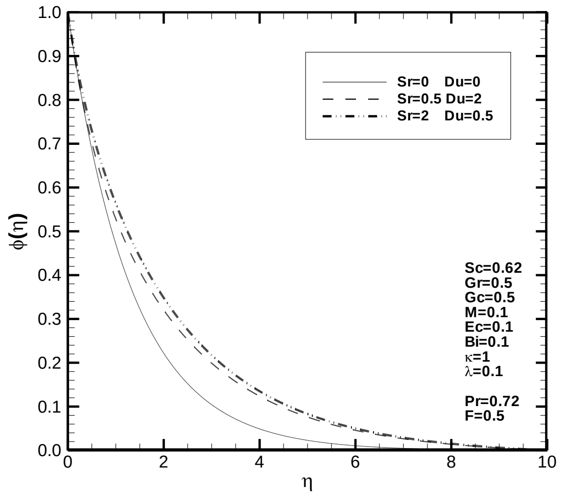

Fig. 12 Effect of Soret and Dufour on the concentration profiles.

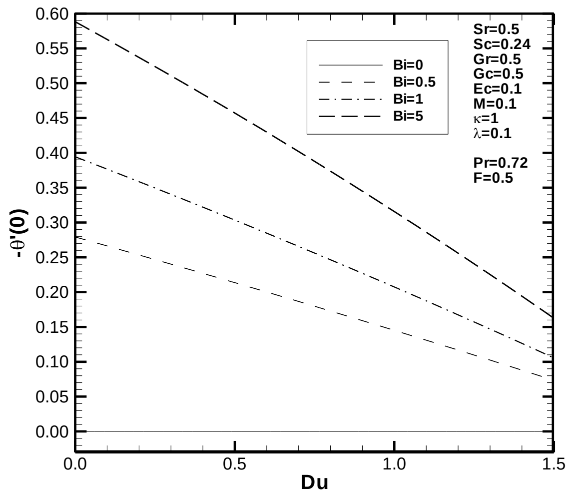

Fig. 14 Variations in $-\theta'(0)$ with Du for various values of Bi.

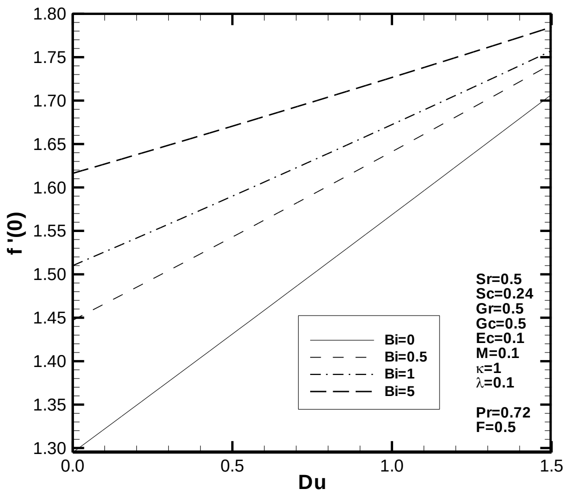

Fig. 13 Variations in $f'(0)$ with Du for various values of Bi.

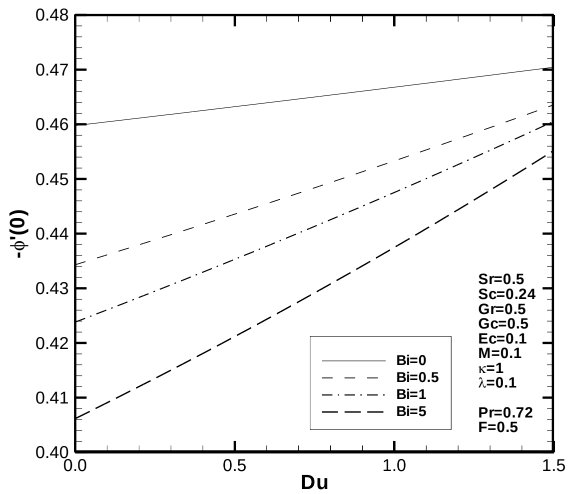

Fig. 15 Variations in $-\phi'(0)$ with Du for various values of Bi.

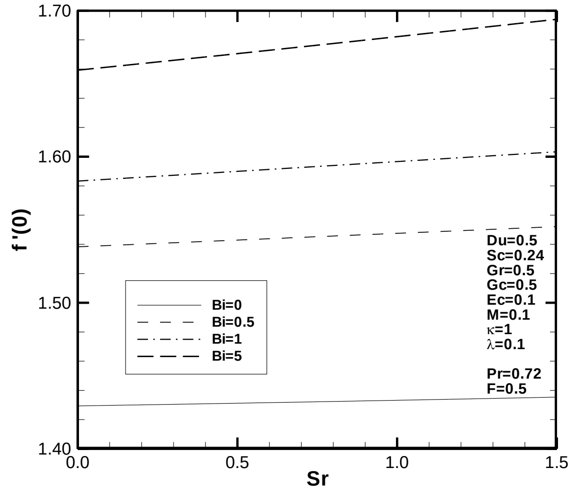

Fig. 16 Variations in $f'(0)$ with Sr for various values of Bi.

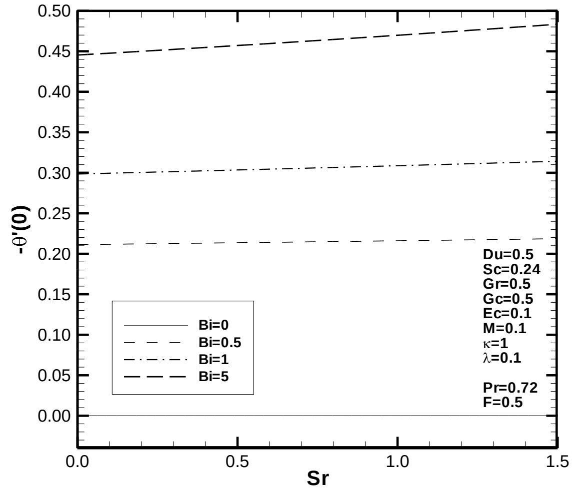

Fig. 17 Variations in $-\theta'(0)$ with Sr for various values of Bi.

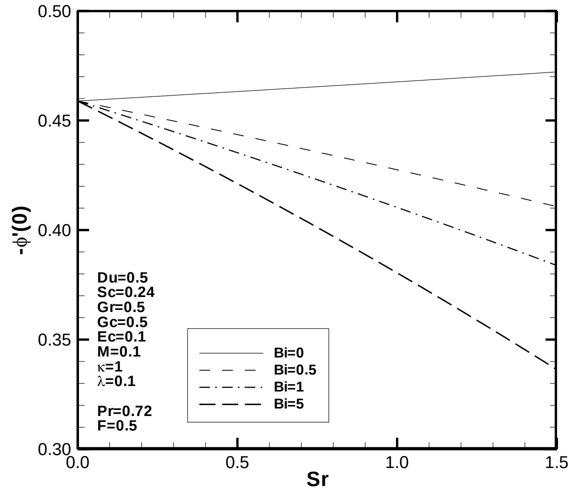

Fig. 18 Variations in $-\phi'(0)$ with Sr for various values of Bi.

equals to zero, the left side of the plate is completely insulated and the Sherwood number tend to increase with increase in the Soret number. It can be also seen from Fig. 18 that the Sherwood number decreases with increasing the Biot number.

5. Conclusions

In this paper, the effects of the Soret and Dufour on MHD mixed convection over an infinite vertical plate with the convective boundary condition at its surface have been studied. Numerical Results shows that the velocity, temperature and concentration increase with increasing the Soret, Dufour and Biot numbers. The effect of Dufour on the velocity and temperature is more than the Sorect effect. The skin-friction coefficient and Sherwood number increase with Dufour number whereas reverse effect is seen on the Nusselt number. Additionally, the Sherwood number decreases with the Soret number while there is no considerable change in the skin-friction coefficient and Nusselt number.

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